

# Conformal Mapping: Schwarz-Christoffel Method for Flux-Switching PM Machines

Ilhan E., Motoasca E., Paulides J., and Lomonova E.

Eindhoven University of Technology  
Den Dolech 2, 5612 AZ Eindhoven, The Netherlands  
e.ilhan@tue.nl

**Abstract** — Novel flux-switching permanent magnet (FSPM) machines with their high energy density are very suitable for electric/hybrid traction applications. Due to the non-periodical and double salient machine structure, it is rather difficult and time consuming to create fast analysis tools. With this aim, different scripts have been written based on the numerical Schwarz-Christoffel (SC) method. Using orthogonal field diagrams (OFD) theory, the magnetic vector potential distribution in the double salient airgap is obtained. Besides using SC as a stand-alone method, integration with other modeling techniques, i.e. tooth contour method (TCM), and finite element method (FEM) are presented. Finally, a comparison is presented for the integrated methods.

**Index terms** — flux-switching machines, orthogonal field diagrams, Schwarz-Christoffel, tooth contour method.

## I. INTRODUCTION

Until recently, the major part in e-mobility solutions has been focused more on ICT (Information and Communication Technologies) and on material development rather than the traction machines. The design of a novel machine structure is a challenging task considering the multi-physical application comprising of electrical, magnetic, mechanic and thermal subgroups. An electromagnetic motor design cannot be achieved simply by using out-of-shelf numerical software or by applying conventional machine knowledge without a proper mathematical model. An improper design will not only affect the motor operation, but the total power-train efficiency.

Flux-switching permanent magnet (FSPM) machines come with many theoretical advantages [1], but they have a rather unconventional structure by embodying all its energy sources in same frame (stator). The geometry, double salient structure, is a must for the machine operation, but it results in a non uniform airgap. All mentioned peculiarities require alternative numerical techniques, which can be in the form of solely numerical or both analytical and numerical, i.e. integrated, techniques. Researchers in this field rely generally on analytical methods [2-3], or only on the finite element method. To understand energy conversion mechanism of the FSPM and to have accurate field results, integrated methods are very effective [4].

In this paper, the main focus is set on the different uses of the Schwarz-Christoffel (SC) theorem. Not only its conformal mapping feature is used for permeance calculations, but also with orthogonal field diagram theory (OFD), the magnetic and electric field distribution in the double salient airgap can be obtained directly. For the machine simulation, the mapping feature is integrated with the tooth contour method (TCM) and results are compared to previous work in [8], where TCM was integrated with FEM. All implementations take place on the 12/10 rotary FSPM structure in Fig.1.

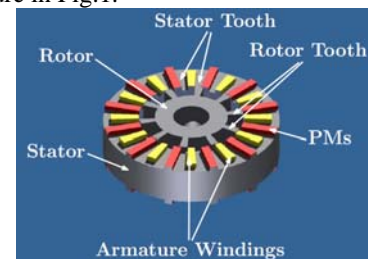


Fig. 1 : 3D illustration of a FSPM with 10 rotor and 12 stator poles

## II. ORTHOGONAL FIELD THEORY (OFD) AND SCHWARZ-CHRISTOFFEL MAPPING (SC)

Orthogonal field diagrams (OFD) consist of orthogonal electric potential and magnetic flux lines extending over the axial length of the machine [5]. Using the widely available MATLAB toolbox for SC-mapping, these diagrams can be simplified to 2-D ones as seen in Fig 2a on a basic double salient machine structure. In the created script, all regions can be changed automatically both in mesh density and in geometrical dimensions. The analysis can be applied from very simple slotless structures without energy sources up to the slotted structures with PMs and windings. The magnetic field distribution just below the upper tooth region is shown in Fig.2b-c.

## III. TOOTH CONTOUR METHOD (TCM) AND SCHWARZ-CHRISTOFFEL MAPPING (SC)

Tooth contour method (TCM) combines the analytical method, magnetic equivalent circuit (MEC) with a numerical technique, i.e. finite element method (FEM), boundary element method (BEM), etc. Where traditional MEC is not efficient, TCM allows integration with accurate modeling techniques for complex topologies [6-7]. In a

double salient structure (e.g. FSPM), analytical MEC lacks in accuracy for calculation of airgap permeances, which are approximated geometrically based on magnetic flux paths. On the contrary, with the TCM these permeances can be calculated by more accurate numerical methods. In previous work in [8], as the numerical method electrostatic FEM (eFEM) was chosen. In this paper, the SC mapping is taken as the alternative to FEM to extract the airgap permeances. How to combine the permeance results from SC into TCM, is given in the flowchart in Fig.3.

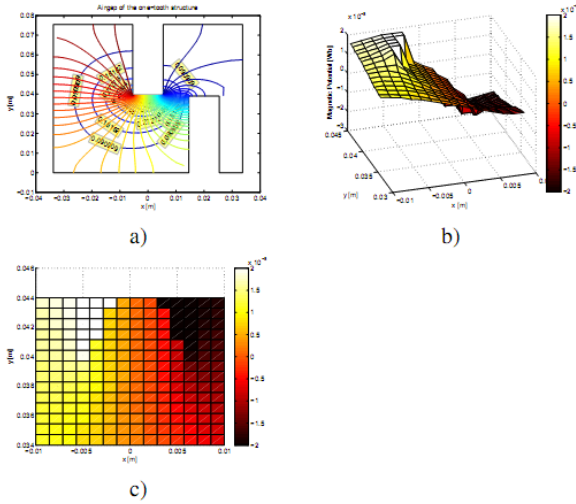


Fig. 2: a) SC mapping, b) magnetic vector potential calculation in the given grid for rotor position in 20mm, c) x-y view of b).

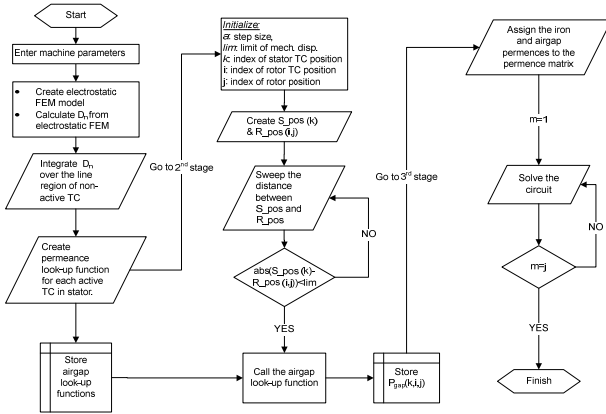


Fig. 3. Flowchart of the MATLAB script for TCM.

#### IV. COMPARISON

On a one-to-one comparison, none of the methods alone gives a satisfactory basis for the FSPM design. TCM's accuracy is more improved than its predecessor MEC but it still lags behind FEM. However, building FEM models can be time consuming. SC gives the possibility for a full integration in MATLAB environment, where a second software program for FEM is not required anymore. Various comparison results are compared in Fig. 4.

Combining OFD with SC, provides a visual environment similar to FEM, next to the magnetic and electric field

calculations. In this implantation, the sources have to be well defined in terms of current sheets or line currents.

#### V. CONCLUSION

This work focused on showing the numerical SC-mapping for the double salient structure of the FSPM. The freely distributed MATLAB toolbox enables calculation of a wide variety of quantities such as magnetic vector potential distribution and magnetic permeance calculations in the non-uniform airgap. This powerful tool, SC, alone or integrated with TCM is capable of both a partial or a complete representation of the FSPM depending on the purpose of the analysis. Results show that this kind of integration of multiple methods is very suitable for SC and can be extended for other machine topologies and modeling techniques.

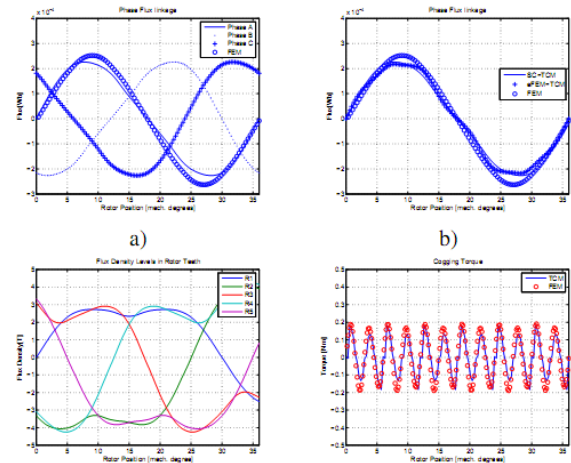


Fig. 4: a) 3-phase flux linkage SC-TCM compared to FEM, b) one phase flux linkage of SC-TCM, eFEM-TCM and FEM, c) flux density levels in rotor teeth calculated by SC-TCM, d) cogging torque calculation of SC-TCM compared to FEM.

#### VI. REFERENCES

- [1] T. Heoung Kim, "A Study on the Design of an Inset-Permanent-Magnet-Type Flux-Reversal Machine," *Magnetics, IEEE Transactions on*, vol.45, no.6, pp.2859-2862, June 2009.
- [2] Krop, D.C.J.; Lomonova, E.A.; Vandenput, A.J.A.;, "Application of Schwarz-Christoffel Mapping to Permanent-Magnet Linear Motor Analysis," *Magnetics, IEEE Transactions on*, vol.44, no.3, pp.352-359, March 2008.
- [3] B.L.J. Gysen, E. Ilhan, K.J. Meessen, J.J.H. Paulides, and E.A. Lomonova, "Modeling of Flux Switching Permanent Magnet Machines With Fourier Analysis," *Magnetics, IEEE Transactions on*, vol.46, no.6, pp.1499-1502, June 2010.
- [4] E. Ilhan, B.L.J. Gysen, J.J.H. Paulides, E.A. Lomonova, "Analytical Hybrid Model for Flux Switching Permanent Magnet Machines," *Magnetics, IEEE Transactions on*, vol.46, no.6, pp.1762-1765, June 2010.
- [5] J. Pyrhonen, T. Jokinen, V. Hrabovcova, Design of Rotating Electrical Machines, 1<sup>st</sup> edition, John Wiley & Sons, Ltd., 2008.
- [6] T. Ramonoso, J.A. Farooq, A. Djerdir, and A. Miraoui, "Reluctance network modeling of surface permanent magnet motor considering iron nonlinearities," vol. 50, pp. 1356-1361, 2009.
- [7] A. Demenko, D. Stachowiak, "Electromagnetic torque calculation using magnetic network methods," *COMPEL*, vol. 27, no. 1, pp17-26, 2008
- [8] E. Ilhan, J. Paulides, L. Encica, E. Lomonova, "Tooth contour method implementation for the flux-switching PM machines," *Electrical Machines (ICEM), 2010 XIX International Conference on*, vol., no., pp.1-6, 6-8 Sept. 2010